**Ay/Ge 117 Bayesian Statistics and Data Analysis**

**2nd Project Check-In**

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Aftershocks are the most prominent expression of the global relaxation process induced by abrupt perturbations of the state of stress in the neighborhood of seismic ruptures. More than 100 years ago, Omori (1894) provided the first quantitative description of an aftershock decay rate, documenting the number of earthquakes triggered by the M8 Nobi earthquake (18 October 1891, Honshu, Japan). To more accurately model the diversity of aftershock decay rates that had been reported later, Utsu (1961) converted the hyperbolic behavior observed by Omori into the so-called modified Omori law (or Omori-Utsu law).

**Update for the dataset:**

The dataset of this class project is acquired from Northern California Earthquake Center. I further select a subset of this dataset so that it covers earthquakes record (time, location, magnitude, etc.) from 1984–present around the location of Loma Prieta. This dataset contains ~4000 earthquakes and will allow me to look at aftershocks occurrence information after the 1989 Loma Prieta earthquake (17 October 1989). In other words, I expect to learn something about the number of aftershocks decay with time, i.e., the Omori-Utsu Law.

**Update for the method:**

The Omori-Utsu law is formulated as:

where is the aftershock frequency within a given magnitude range, t the time from the triggering event (the so-called main shock), K the productivity of the aftershock sequence, p the power law exponent, and c the time delay before the onset of the power-law aftershock decay rate.

There are three unknown parameters here I want to estimate, i.e., K, c, and p.

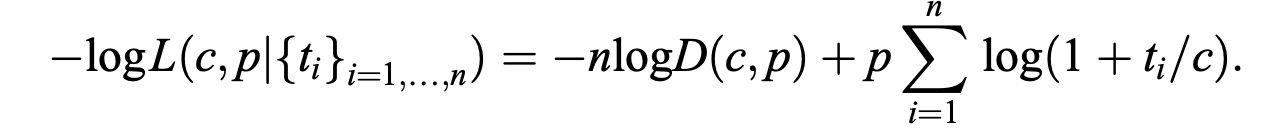
The typical type of Omori-Utsu law is like this figure (after April 25 2015 Nepal Earthquake):

Chart, scatter chart

Description automatically generated

I found a study (Holschneider et al., 2012) applying Bayesian approach to estimate the parameters in the Omori-Utsu aftershock decay law. Holschneider et al. proposed that the three parameters {K, c, p} cannot be estimated independently. The author separated them and suggested to estimate {c, p} in a Bayesian framework. Then K is just depending on the best-fit {c, p}.

The author has derived and proposed a form for the log likelihood function for {c, p}:



The minimum of this function yields the maximum likelihood estimator c\* and p\* for c and p, respectively.

The function D(c, p) is defined as following,

for cases when p ≠ 1:

A picture containing text

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for a specific case when p = 1:

Text

Description automatically generated with medium confidence

Then the parameter K simply depends on the best-fit {c, p}

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where here is the number of earthquakes within the time window (from ***tstart*** to ***tstop***) of our analyzed dataset. So, obtaining this and thus, K, should be trivial.

**Update results:**

Applying the methods mentioned above, I am still figuring out the calculation of the log-likelihood function. My current problem is that the likelihood function plot does not look correct. I’ve been trying both the real dataset (the one at Loma Prieta) and a simple small synthetic dataset (based on some arbitrary assumed parameters {K, c, p}). However, after calculating the log-likelihood across the {c, p} parameter space for both of the dataset, the results seem weird.

This is my synthetic dataset:

Shape

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Below are my log-likelihood images (color coded with log-likelihood):

(real dataset) (synthetic dataset with {K, c, p}={10, 0.1, 1})

Shape, rectangle

Description automatically generatedShape, rectangle

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But a proper result from the original paper should look like:

This is also a synthetic dataset, with p = 1 and c = 0.02 day (the white dot)

A picture containing shape

Description automatically generated

**Reference for methods and dataset:**

Aki, K. (1965). Maximum Likelihood estimate of b in the formula logN = a – bM and its confidence limits, Bull. Earthquake Res. Inst. 43,237 -239.

Holschneider, M., Narteau, C., Shebalin, P., Peng, Z., & Schorlemmer, D. (2012). Bayesian analysis of the modified Omori law. Journal of Geophysical Research: Solid Earth, 117(B6).

Utsu, T. (1961). A statistical study on the occurrence of aftershocks. Geophys. Mag., 30, 521-605.

Waldhauser, F. and D.P. Schaff (2008). Large-scale relocation of two decades of Northern California seismicity using cross-correlation and double-difference methods, J. Geophys. Res.,113, B08311, doi:10.1029/2007JB005479

Waldhauser, F. (2009). Near-real-time double-difference event location using long-term seismic archives, with application to Northern California, Bull. Seism. Soc. Am., 99, 2736-2848, doi:10.1785/0120080294

**Brief information about 1989 Magnitude 6.9, Loma Prieta Earthquake:**

https://www.usgs.gov/natural-hazards/earthquake-hazards/science/m69-october-17-1989-loma-prieta-earthquake?qt-science\_center\_objects=0#qt-science\_center\_objects

https://en.wikipedia.org/wiki/1989\_Loma\_Prieta\_earthquake